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Finite Element Analysis of Flow Induced Vibration in Pipes

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Abstract*-Flow induced vibrations of pipes with internal fluid flow is studied in this work. Finite Element Analysis methodology is used to determine the critical fluid velocity that the threshold of pipe instability. The partial differential equation of motion governing the lateral vibrations of the pipe is employed to develop the stiffness and inertia matrices corresponding to two of the terms of the equations of motion. The Equation of motion further includes a mixed-derivative term that was treated as a source for a dissipative function. The corresponding matrix with this dissipative function was developed and recognized as the potentially destabilizing factor for the lateral vibrations of the pipe which carries the fluid.*

Key Words**:** Flow induced vibration, pipe instability, critical fluid velocity.

1.Introduction

The flow of a fluid through a pipe can impose pressures on the walls of the pipe causing it to deflect under certain flow conditions. This deflection of the pipe may lead to structural instability of the pipe. The fundamental natural frequency of a pipe generally decreases with increasing velocity of fluid flow. There are certain cases where decrease in this natural frequency can be very important, such as very high velocity flows through flexible thin-walled pipes such as those used in feed lines to rocket motors and water turbines. The pipe becomes susceptible to resonance or fatigue failure if its natural frequency falls below certain limits. With large fluid velocities the pipe may become unstable. The study of dynamic response of a fluid conveying pipe in conjunction with the transient vibration of ruptured pipes reveals that if a pipe ruptures through its cross section, then a flexible length of unsupported pipe is left spewing out fluid and is free to whip about and impact other structures. In power plant plumbing pipe whip is a possible mode of failure.

The objective of this work is to implement numerical solutions method, more specifically the Finite Element Analysis (FEA) to obtain solutions for different pipe configurations and fluid flow characteristics. The governing dynamic equation describing the induced structural vibrations due to internal fluid flow has been formed and discussed. The governing equation of motion is a partial differential equation that is fourth order in spatial variable and second order in time. Parametric studies have been performed to examine the influence of mass distribution along the length of the pipe carrying fluid.

Initial investigations on the bending vibrations of a simply supported pipe containing fluid were carried out by Ashley and Haviland[2]. Subsequently, Housner[3] derived the equations of motion of a fluid conveying pipe more completely and developed an equation relating the fundamental bending frequency of a simply supported pipe to the velocity of the internal flow of the fluid. He also stated that at certain critical velocity, a statically unstable condition could exist. Long[4] presented an alternate solution to Housner's[3] equation of motion for the simply supported end conditions and also treated the fixed-free end conditions. He compared the analysis with experimental results to confirm the mathematical model. His experimental results were rather inconclusive since the maximum fluid velocity available for the test was low and change in bending frequency was very small. Other efforts to treat this subject were made by Benjamin, Niordson[6] and Ta Li. Other solutions to the equations of motion show that type of instability depends on the end conditions of the pipe carrying fluid. If the flow velocity exceeds the critical velocity pipes supported at both ends bow out and buckle[1]. Straight Cantilever pipes fall into flow induced vibrations and vibrate at a large amplitude when flow velocity exceeds critical velocity[8-11].

2. Formulation of Problem

In this Section, a mathematical model is formed by equations of a straight fluid conveying pipe and these equations are later solved for the natural frequency and onset of instability of a cantilever and pinned-pinned pipe.

2.1 Finite Element Model

Consider a pipe of length L, modulus of elasticity E, and its transverse area moment I. A fluid Flows through the pipe at pressure p and density ρ at a constant velocity v through the internal pipe cross-section of area A. where the mass per unit length of the pipe and the fluid in the pipe is given by $M = m + \rho A$. We have the equation of motion for free vibration of a

fluid carrying pipe as,

$$EI\frac{\partial^4 Y}{\partial x^4} + \rho AV^2\frac{\partial^2 Y}{\partial x^2} + 2\rho AV\frac{\partial^2 Y}{\partial x\,\partial t} + M\frac{\partial^2 Y}{\partial t^2} = 0. \quad (1)$$

2.1.1 Formulating the Stiffness Matrix for a Pipe Carrying Fluid

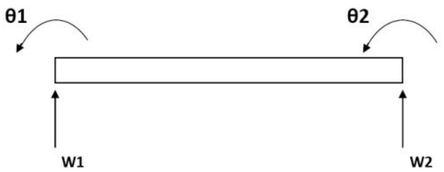

Figure 1. Beam Element Model

The element stiffness matrix for the beam is,

$$[K_1]^e = EI/\ell^3 \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

(2)

2.1.2 Forming the Matrix for the Force that conforms the Fluid to Flow through the Pipe

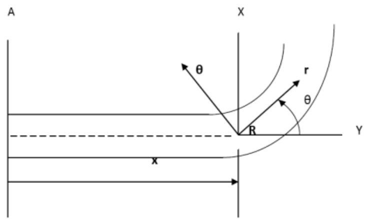

Figure 2. Pipe Carrying Fluid

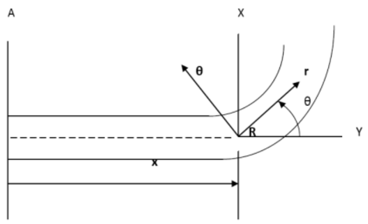

Figure 2. Pipe Carrying Fluid

Consider a pipe carrying fluid and let R be a point at a distance x from a reference plane AB as shown in figure 5. Due to the flow of the fluid through the pipe a force is introduced into the pipe causing the pipe to curve. The elemental matrix for the force.

$$[K_2]^e = \rho A V^2/30\ell \begin{bmatrix} 36 & 3 & -36 & 3 \\ 3 & 4 & -3 & -1 \\ -36 & -3 & 36 & -3 \\ 3 & -1 & -3 & -4 \end{bmatrix}$$

(3)

2.1.3 Dissipation Matrix Formulation for a Pipe carrying Fluid

The dissipation matrix represents the force that causes the fluid in the pipe to whip creating instability in the system.

$$[D]^e = \rho A V/30 \begin{bmatrix} 36 & 3 & -36 & 3 \\ 3 & 4 & -3 & -1 \\ -36 & -3 & 36 & -3 \\ 3 & -1 & -3 & -4 \end{bmatrix}$$

(4)

2.1.4 Inertia Matrix Formulation for a Pipe carrying Fluid

The elemental mass matrix for a pipe.

$$[M]^e = M\ell/420 \begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix}$$

(5)

3. Finite Element Analysis of Flow Induce Vibration in Pipes

3.1 Global stiffness matrix

$$[K]_{Global} = EI/\ell^3$$

$$\begin{bmatrix} 12 & 6l & -12 & 6l & 0 & 0 \\ 6l & 4l^2 & -6l & 2l^2 & 0 & 0 \\ -12 & -6l & (12+12) & (-6l+6l) & -12 & 6l \\ 6l & 2l^2 & (-6l+6l) & (4l^2+4l^2) & -6l & 2l^2 \\ 0 & 0 & -12 & -6l & 12 & -6l \\ 0 & 0 & 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

(6)

3.2 Applying Boundary Conditions to Global Stiffness Matrix for simply supported pipe with fluid flow

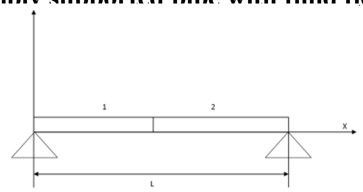

Figure 3. Representation of Simply Supported Pipe Carrying Fluid.

$$[K]_{Globals} = EI/\ \ell^3 \begin{bmatrix} 4l^2 & -6l & 2l^2 & 0 \\ -6l & (12+12) & (-6l+6l) & 6l \\ 2l^2 & (-6l+6l) & (4l^2+4l^2) & 2l^2 \\ 0 & 6l & 2l^2 & 4l^2 \end{bmatrix}$$

(7)

3.3 Applying Boundary Conditions to Global Stiffness Matrix for a cantilever pipe with fluid flow

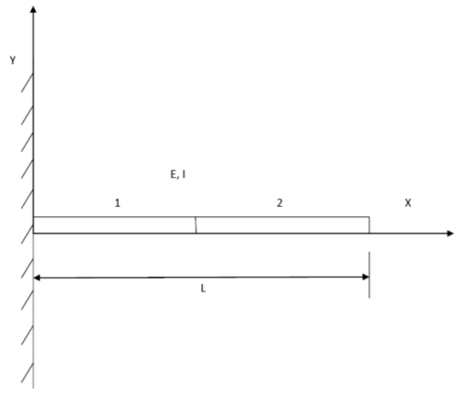

Figure 4. Representation of Cantilever Pipe Carrying Fluid

When the boundary conditions are applied to a Cantilever pipe carrying fluid, the Global Stiffness Matrix as follows

$$[K]_{Globals} = EI/\ \ell^3 \begin{bmatrix} (12+12) & (-6l+6l) & -12 & 6l \\ (-6l+6l) & (4l^2+4l^2) & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ -6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

(8)

4. Assembling Global Matrices for Simply Supported and Cantilever pipe carrying fluid

The expression for natural frequency and critical velocity of the simply supported pipe carrying fluid is given by;

$$wn = \frac{(3.14)^2}{l^2} * \sqrt{E * I/M}$$

(9)

$$vc = \frac{3.14}{L} * \sqrt{E * I/\rho A} \quad (10)$$

The expression for natural frequency and critical velocity of the simply supported pipe carrying fluid is given by;

$$wn = \frac{(1.875)^2}{L^2} * \sqrt{E * I/M} \quad (11)$$

$$vc = \frac{1.875}{L} * \sqrt{E * I/\rho A} \quad (12)$$

A MatLab Program is created to assemble the global stiffness matrices. The number of elements, density, length, modulus of elasticity of the pipe, density and velocity of fluid flowing through the pipe and the thickness of the pipe can be defined by the user. The Results from Matlab Program are shown in subsequent sections.

4.1 Parametric Study of flow Induced Vibrations in Pipes

The study is carried out on a single span steel pipe with a 0.01 m (0.4 in) diameter and a 0.0001 m (0.004 in) thick wall.

Table 1. The other parameters of the Fluid and Pipe

Density of the Pipe ρ_p (Kg/m^3)	**Density of the Fluid ρ_f (Kg/m^3)**	**Length of the Pipe L(m)**	**Number of Elements n**	**Modulus of Elasticity E (Gpa)**
8000	1000	2	10	207

The fundamental frequency of vibration and the critical velocity of fluid for a simply supported pipe carrying fluid are, wn = 21.8582 rad/sec, vc = 16.0553 m/sec.

5. Results and Analysis

Table 2. Reduction of Fundamental Frequency for a Pinned-Pinned(Simply Supported) Pipe with increasing Flow Velocity

Velocity of Fluid (v) m/sec	Velocity Ratio(v/v_c)	Frequency(w) rad/sec	Frequency Ratio(w/w_n)
2	0.1246	21.5619	0.9864
6	0.3737	18.8644	0.8630
10	0.6228	12.1602	0.5563
14	0.8720	0.3935	0.0180
16.0553	1	0	0

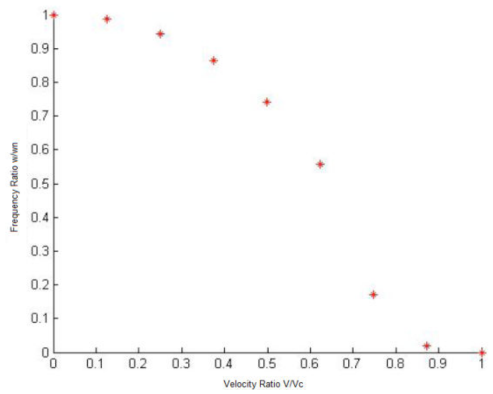

Figure 5. Reduction of Fundamental Frequency for a Pinned-Pinned (Simply Supported) Pipe with increasing Flow Velocity.

The fundamental frequency of vibration and the critical velocity of fluid for a Cantilever pipe carrying fluid are, wn = 7.7940 rad/sec, vc = 9.5872 m/sec.

Table 3. Reduction of Fundamental Frequency for a Pinned-Free(Cantilever) Pipe with increasing Flow Velocity

Velocity of Fluid (v) m/sec	Velocity Ratio(v/v_c)	Frequency(w) rad/sec	Frequency Ratio(w/w_n)
2	0.2086	7.5968	0.9747
4	0.4172	6.9807	0.8957
6	0.6258	5.8549	0.7512
8	0.8344	3.8825	0.4981
9	0.9388	1.9897	0.2553
9.5872	1	0	0

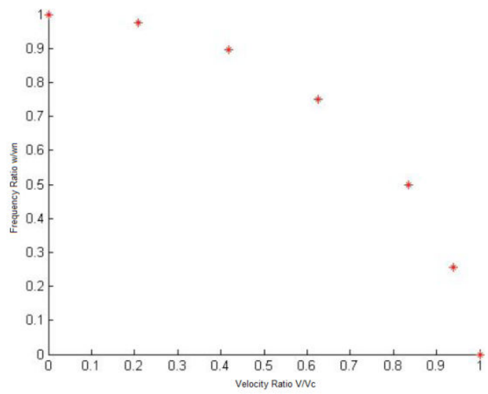

Figure 6. Reduction of Fundamental Frequency for a Pinned-Free (Cantilever)Pipe with increasing Flow Velocity.

As the Fluid Flow Velocity increases at a certain Velocity of Fluid Fundamental Frequency tends to Zero for Simply Supported as well as for Cantilever Pipe.

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